(for either variable pressure injection, variable area injection, or any combination of the two) if the schedule of required vehicle turning moment vs time is known and if a subscale data point is available. This expression has been used extensively with Freon-12, perchloroethylene, Freon-113, and Freon-114B-2. The subscale conditions must simulate the full scale conditions insofar as injection angle, number of orifices (and geometrical arrangement), axial location, and nozzle half angle are concerned.

Effect of Injectant Characteristics

The total side-force resulting from the injection of a fluid into the expansion cone of a rocket nozzle may be considered to consist of two components: 1) a component of side-force due to the thrust of the fluid upon injection (product of mass flow rate and velocity plus a pressure area term for gas injection), and 2) a component due to the interaction of the injected fluid with the mainstream gases. The second component consists of a static pressure recovery of the mainstream gases acting over an asymmetrical area within the nozzle. The pressure area force resulting from this pressure recovery represents from 80 to 90% of the total developed side-force in the case of liquid injection.

A one-dimensional model of this fluid interaction process has been analyzed.[8] The results of this analysis indicate that the injectant should provide as large an obstruction as possible to mainstream flow and should react or decompose with a release of heat or, in the case of an inert fluid, vaporize and/or dissociate with a minimum amount of heat absorption. Consequently, the following, and in some cases conflicting, injectant characteristics are desired: 1) low specific heat in liquid and vapor phases, 2) low boiling point, 3) low heat of vaporization, 4) high heat of reaction or exothermic decomposition, 5) low molecular weights of products of combustion or decomposition, and 6) high density (from a packaging standpoint).

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Shock-Induced Boundary Layer Separation in Overexpanded Conical Exhaust Nozzles

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The flow in overexpanded supersonic nozzles is reviewed. Although five essentially different flow regimes can be discerned, depending on the nozzle pressure ratio, the regime of most interest to the engine designer is the one characterized by oblique shock patterns in the nozzle and flow separation from the nozzle wall. It is shown that the pressure rise associated with the separation correlates well with the Mach number at the separation point. A simple analytical formulation for the pressure rise required to separate the flow provides excellent agreement with experimental data over a wide range of nozzle operating conditions and allows prediction of overexpanded nozzle performance.

Nomenclature

M = Mach number
p = pressure
u = velocity
α = nozzle half angle
γ = specific heat ratio
δ = flow deflection angle

Subscripts and superscripts

a = ambient
b = origin of separated region
c = stagnation at nozzle entry
s = origin of shock wave-boundary layer interaction
$*$ = characteristic velocity

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Introduction

THE performance of fully expanded and underexpanded conical exhaust nozzles, having conventional divergence angles, can be described adequately on the basis of one-dimensional fluid mechanics. Even though a one-dimensional model of nozzle flow can be postulated for overexpanded nozzles, it is now well known that it generally does not correspond to reality. Whereas for the one-dimensional model it is assumed that the boundary condition of ambient pressure at the nozzle exit is satisfied by normal shock deceleration followed by subsonic compression within the nozzle,[1] in actuality the flow may attain ambient pressure as the result of oblique shock compression accompanied by flow separation from the wall.

Overexpanded conical nozzle performance may be divided into the following five distinct regimes, in order of increasing nozzle pressure ratio:

1) *Essentially one-dimensional flow with a normal shock downstream of the throat.* This condition occurs for pressure ratios slightly above those required to choke the nozzle. (For convergent-divergent nozzles, this presure ratio is less than that required to choke a convergent nozzle.)

2) *Unstable flow with oblique shock boundary layer separation.* The shock pattern is asymmetric and unsteady.

3) *Stable flow with symmetric oblique shock boundary layer separation around the nozzle circumference.* As the pressure ratio is raised, the separation point moves toward the nozzle exit. This regime exists from a nozzle pressure ratio sufficient to establish stable flow until the pressure ratio is raised to the point where separation occurs in the immediate proximity of the nozzle exit plane.

4) *Flow with oblique shock boundary layer interaction within the nozzle but in the immediate proximity of the nozzle exit plane.* This regime exists until the pressure ratio is sufficiently high to expel the oblique shock pattern from the nozzle.

5) *Undisturbed flow in the nozzle with oblique shocks initiating in the nozzle exit plane.* The shocks become weaker as the nozzle pressure ratio is raised and disappear when the full expansion pressure ratio is reached.

For rocket motors and jet engines, nozzle geometry and operating pressure are usually such that only regimes 3–5 are of interest to the engine designer. Nozzle performance in regime 5 can be estimated adequately on the basis of one-dimensional fluid mechanics while correcting for velocity deviations from axial in the exit plane.[2] Regime 4 covers only a very small range of pressure ratios. Since the shock wave boundary layer interaction is restricted to the immediate vicinity of the nozzle exit plane, nozzle performance, except for base drag effects, can be estimated as for regime 5. Estimating nozzle performance for regime 3 requires knowledge of the location of the boundary layer separation.

A number of experimental investigations on conical and straight-walled two-dimensional nozzles have been carried out in order to determine the conditions for which flow separation takes place and the location of the separation points.[3–12] Within the scatter of the experimental data for the ratio of the wall pressure at the separation point to the ambient pressure, there seems to be no consistent significant difference between two-dimensional and axisymmetric nozzles, nor can any trend be detected as a function of nozzle half angle, for a range of half angles between 7° and 30°. Fraser et al.,[6] McKenney,[4] and Foster and Cowles[3] conclude that flow separation occurs at the point where the nozzle wall pressure reaches a particular fraction of the ambient pressure. The "Summerfield criterion" predicts separation for $p_s/p_a < 0.4$.[13] An alternate attempt to predict the separation point is based on the assumption that the oblique shock causing separation is such as to turn the flow at the wall to the axial direction.[5]

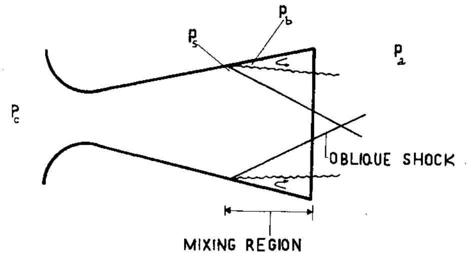

Fig. 1 Overexpanded nozzle with separated flow

Separation of the Nozzle Boundary Layer

The shock wave boundary layer interaction resulting in separation of the flow from the nozzle wall (Fig. 1) may be divided into the initial region of compression terminating at the point of separation and the subsequent mixing region wherein the separated nozzle flow mixes with entrained air from the atmosphere. Whereas by far the major pressure rise occurs up to the separation point, a small amount of compression accompanies the subsequent mixing. Although no detailed investigation of the flow in the mixing region seems to have been made, it would seem that the pressure rise occurring in this region is dependent on the Mach number of the separated jet, as well as on the geometry of the separated region. Examination of a number of wall pressure distributions in separated nozzle flow indicates that the pressure rise in the mixing region generally constitutes 3 to 10% of the pressure rise from the beginning of the interaction until ambient pressure is attained.[14]

It is important to note that rapid compression to the separation point and subsequent mixing are inherent aspects of turbulent shock wave boundary layer separations as observed on forward-facing steps, compression surfaces, and incident shock models.[15, 16] In these interactions, however, the pressure rise accompanying the mixing stage is often fairly large, and mixing is terminated by further compression as the flow reattaches. A review of the experimental data contained in Refs. 15 and 16 indicates that within the accuracy of the data the pressure rise to separation is independent of the geometry of the model, depending only on the freestream Mach number. If these observations are interpreted as indicating that a turbulent supersonic boundary layer separates when subjected to a critical pressure jump, whose value depends only on the Mach number and not on the geometry of the interaction, it is to be expected that separation in supersonic exhaust nozzles should occur for pressure jumps of the same magnitude. Lower pressure jumps are not capable of forcing separation, whereas separation with pressure jumps larger than critical would not be stable. The pressure rise accompanying separation in exhaust nozzles is therefore a function of the stream Mach number at the separation point.

Recent Measurements of Nozzle Separation Pressures

Most of the experimental investigations on the basis of which separation criteria were formulated have been limited to nozzles operating at high pressure ratios with Mach numbers at the separation point between 2.5 and 3.5. In order to ascertain the Mach number dependence of the nozzle separation pressure ratio, the present authors measured the separation pressure ratios of overexpanded nozzles when operating at low pressure ratios.[17] The results of this investigation are plotted together with other available nozzle separation data in Fig. 2. The separation to ambient pressure ratio is plotted against the separation Mach number computed on the basis of isentropic expansion from the nozzle inlet stagnation pressure to the wall pressure at the separation point. The data represent two-dimensional and conical nozzles, nozzle half angles between 7° and 30°, and gas specific heat ratio between 1.2 and 1.4.

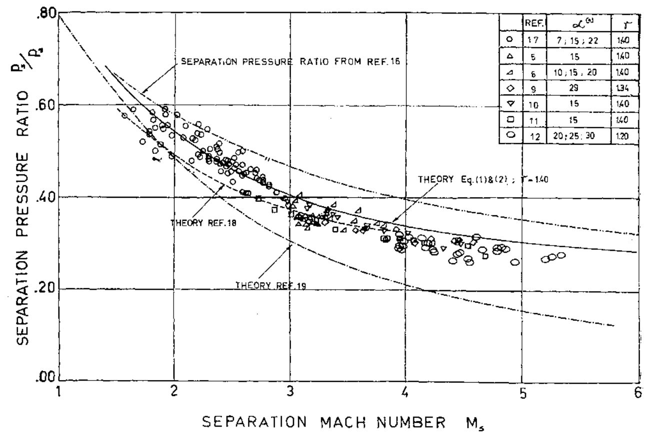

Fig. 2 Nozzle separation pressure ratio

Although the pressure ratio shown in Fig. 2 is not the true pressure ratio to separation, but rather includes also the additional pressure rise occurring in the mixing region, the data show a very clear and consistent variation with the separation Mach number. Also shown in Fig. 2 is a curve taken from Ref. 16 representing the separation pressure ratio as measured on forward-facing steps, compression surfaces, and incident shock models. It can be seen that the trend of these data is identical to that of the nozzle separation data. Moreover, if it is recalled that the nozzle separation pressure ratio shown includes the pressure rise for the mixing region, it seems that the separation pressure rise is the same within the scatter of the data from both types of investigation. The data represent a wide range of Reynolds numbers. There does not seem to be a significant effect of Reynolds number on the separation pressure ratio.

The pressure rise associated with separations occurring in the immediate vicinity of the nozzle exit plane (regime 4) generally does not correlate well with the rest of the data. It may be surmised that in such cases the geometry of the interaction, and in particular the circulation of ambient air in close proximity to the separation point, does affect the pressure rise required to force separation.

Separation Pressure Ratio

Some of the theories advanced to predict turbulent shock wave-boundary separation have been used by their authors to attempt to predict separation in supersonic nozzles. Theoretical estimates of separation pressure ratios for nozzle flow have been published by Mager[18] and Page.[19, 20] Application of their theories to separated nozzle flow implies the assumption that the pressure ratio required to separate a turbulent boundary layer is independent of the geometry of the interaction. Furthermore, the effect of the pressure gradient and of boundary layer history also is neglected. Examination of experimental data for the separation pressure ratio for a number of model configurations as well as for nozzle flow indicates that these assumptions generally are justified.

Best agreement with the experimental data for the shock pressure ratio required to separate a turbulent boundary layer is obtained by using the assumption that the pressure rise must be sufficient to stagnate a characteristic velocity u_s^* in the boundary layer. This simple approach seems to have been suggested first by Gadd.[21] Assuming a constant stagnation temperature in the boundary layer, the characteristic Mach number is given by

$$M_s^* = \frac{M_s(u_s^*/u_s)}{\{1 + [(\gamma - 1)/2]M_s^2[1 - (u_s^*/u_s)^2]\}^{1/2}} \qquad (1)$$

For $M_s^* < 1$, it is assumed that stagnation occurs isentropically. The required pressure ratio is

$$\frac{p_b}{p_s} = \left\{\frac{1 + [(\gamma - 1)/2]M_s^2}{1 + [(\gamma - 1)/2]M_s^2[1 - (u_s^*/u_s)^2]}\right\}^{(\gamma/\gamma - 1)} \qquad (2)$$

If for $M_s^* > 1$ it is assumed that isentropic stagnation is preceded by normal shock compression, the required pressure ratio is

$$\frac{p_b}{p_s} = \frac{\left[\dfrac{\gamma + 1}{2} M_s^2 \left(\dfrac{u_s^*}{u_s}\right)^2\right]^{\gamma/(\gamma - 1)}}{\left\{1 + \dfrac{\gamma - 1}{2} M_s^2 \left[1 - \left(\dfrac{u_s^*}{u_s}\right)^2\right]\right\}\left\{\dfrac{M_s^2}{2}\left[(\gamma + 1)\left(\dfrac{u_s^*}{u_s}\right)^2 - \dfrac{(\gamma - 1)^2}{\gamma + 1}\right] - \dfrac{\gamma - 1}{\gamma + 1}\right\}^{1/(\gamma - 1)}} \qquad (3)$$

On comparing p_s/p_b, as computed from Eqs. (1) and (2), with the available nozzle separation data for p_s/p_a, very good agreement is obtained for $u_s^*/u_s = 0.6$. The agreement between the theory and experiment can be noted on Fig. 2, where the predictions of Refs. 18 and 19 also are shown. It should be borne in mind that none of the theories account for the compression associated with the mixing region. As regards the theory presented in this paper, it is likely that the true value of u_s^*/u_s is smaller than 0.6. A value of 0.56 leads to very good agreement with the experimental data for steps, compression surfaces, and incident shock models.

If the flow in the nozzle up to the separation point is assumed isentropic, then the separation Mach number is given by

$$M_s^2 = [2/(\gamma - 1)][(p_c/p_a)^{(\gamma-1)/\gamma}(p_a/p_s)^{(\gamma-1)/\gamma} - 1] \qquad (4)$$

Neglecting the pressure rise in the mixing region, i.e., $p_b = p_a$, and substituting Eq. (4) into Eq. (2) or (3) provides an equation for the ratio of the separation pressure to ambient pressure as a function of the nozzle pressure ratio. Using Eq. (2) for $M_s^* < 1$, the separation pressure ratio is obtained explicitly as

$$\frac{p_s}{p_a} = \frac{p_c/p_a}{\left[\dfrac{(p_c/p_a) - (u_s^*/u_s)^2}{1 - (u_s^*/u_s)^2}\right]^{\gamma/(\gamma - 1)}} \qquad (5)$$

For Mach numbers lower than about 1.13, the pressure jump required to separate the boundary layer is larger than the pressure ratio provided by normal shock compression. Therefore, for nozzle pressure ratios sufficiently low so that normal shock compression at $M < 1.13$ followed by subsonic diffusion provides for ambient pressure at the nozzle exit plane, classical overexpanded nozzle flow (regime 1) will occur. The nozzle pressure ratio necessary for oblique shock boundary layer separation at a Mach number of 1.13 (assum-

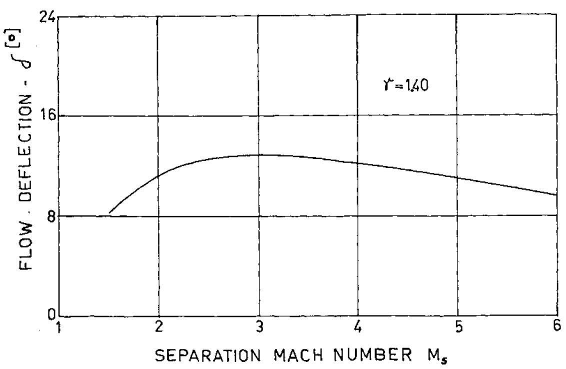

Fig. 3 Flow deflection for two-dimensional nozzle

ing $u_s^*/u_s = 0.6$ and $\gamma = 1.4$) is 1.7 as computed from Eq. (2), neglecting the pressure rise due to mixing. It is unlikely that oblique shock separation will occur for nozzle pressure ratios lower than 1.7. For nozzles with very small exit to throat area ratios, normal shock flow can be obtained for nozzle pressure ratios up to 1.89, the limiting value corresponding to ideally expanded convergent nozzle flow. For significant area ratios, however, oblique shock flow will be established for pressure ratios exceeding 1.7. It should be noted that the nozzle pressure ratio required for flow, characterized by a normal shock at the limiting Mach number followed by subsonic compression, is lower than the nozzle pressure ratio required for separated flow at the limiting Mach number. Whether a stable flow configuration exists for the intermediate pressure ratios remains to be determined.

The flow deflection corresponding to a two-dimensional oblique shock of strength equal to the separation pressure ratio as given by Eqs. (2) and (3) is shown in Fig. 3. It is seen that for conventional nozzle divergence angles the flow generally would be turned to within a few degrees of axial.

Conclusions

The pressure jump associated with shock wave-boundary layer separation in overexpanded nozzles is to a good approximation the same as the separation pressure rise observed in supersonic turbulent flow on forward-facing steps, compression surfaces, and incident-shock models. Nozzle separation therefore can be predicted by using data for the separation pressure ratio as a function of the separation Mach number. Very good correlation with experimental nozzle data is obtained if it is assumed that separation occurs when the pressure rise is sufficient to arrest a characteristic streamline in the boundary layer whose velocity is 0.6 of the freestream velocity. For subsonic characteristic streamline velocities, stagnation is assumed isentropic, whereas for supersonic velocities it is assumed that isentropic stagnation is preceded by normal shock compression.

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